LECTURE 5: CONTINUOUS MODELS

Modeling and Simulation 2

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POWER SYSTEM DAEs

- Separation of time scales
- Modular Design

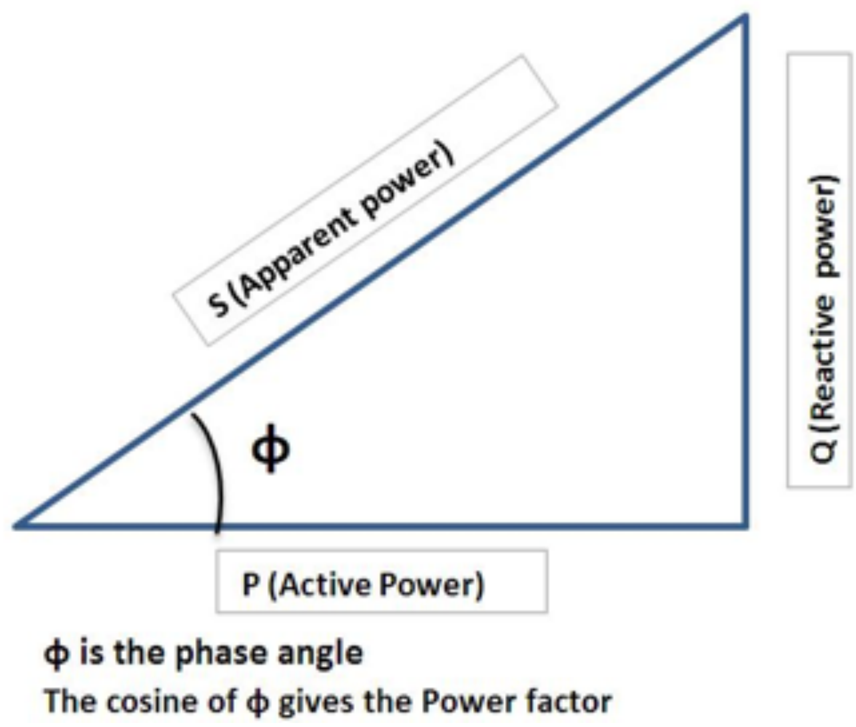
AC CIRCUITS

Phasor voltage is related to phasor current by

$$V_{ab} = I_{ab} Z_{ab}$$

Phasor expressions for voltage and current can be used to derive an expression for phasor power.

$$S = P + iQ = VI^*$$



PER UNIT NOTATION

Power transmission lines are operated at voltage levels in the ranges of kV.

For convenience, quantities are often expressed as "per unit" of a base value.

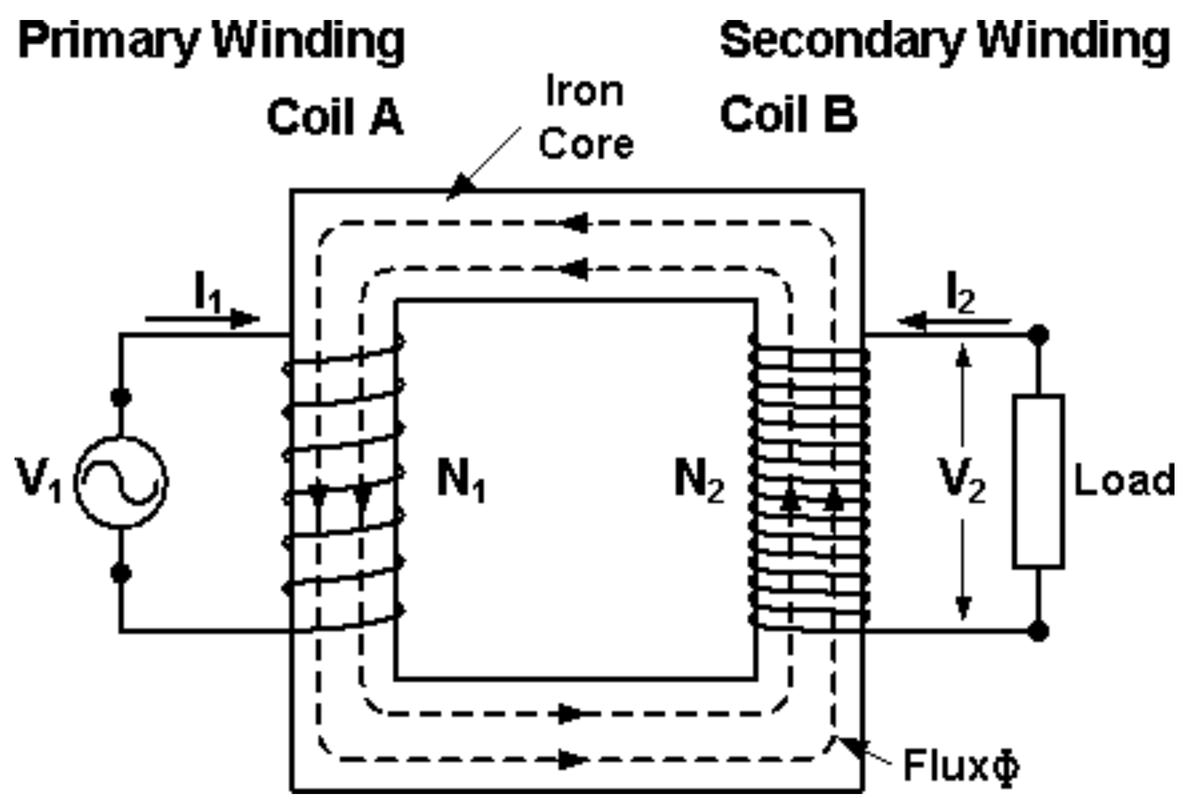
Kirchhoff's current and voltage laws applied in per unit scale return per unit quantities.

$$\text{Base current, A} = \frac{\text{base, kVA}}{\text{base voltage, kV}}$$

$$\text{Base impedance, } \Omega = \frac{\text{base voltage, V}}{\text{base current, A}}$$

$$\text{Per-unit impedance/voltage/current} = \frac{\text{actual impedance/voltage/current}}{\text{base impedance/voltage/current}}$$

TRANSFORMERS



Assumptions:

1) permeability is infinite

2) no flux leaked

3) no winding resistance

4) no losses in iron core

$$v_1 = e_1 = N_1 \frac{d\phi}{dt}$$

$$v_2 = e_2 = N_2 \frac{d\phi}{dt}$$

$$\frac{V_1}{V_2} = \frac{E_1}{E_2} = \frac{N_1}{N_2}$$

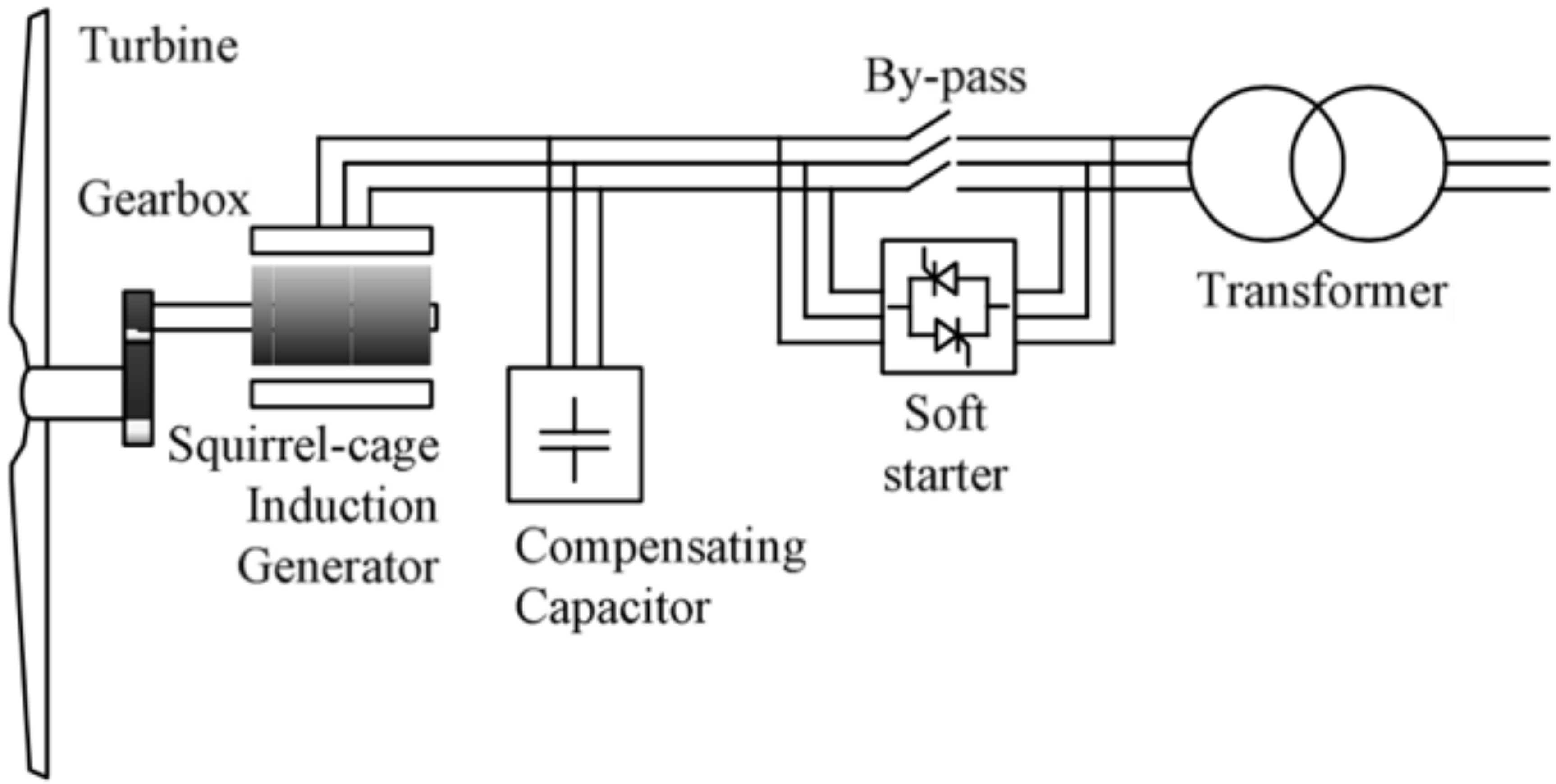
NETWORK

$$0 = \frac{n}{Z_1}(V_0 - nV_2) - I_S - I_C - \frac{1}{Z_2}(V_2 - V_3)$$

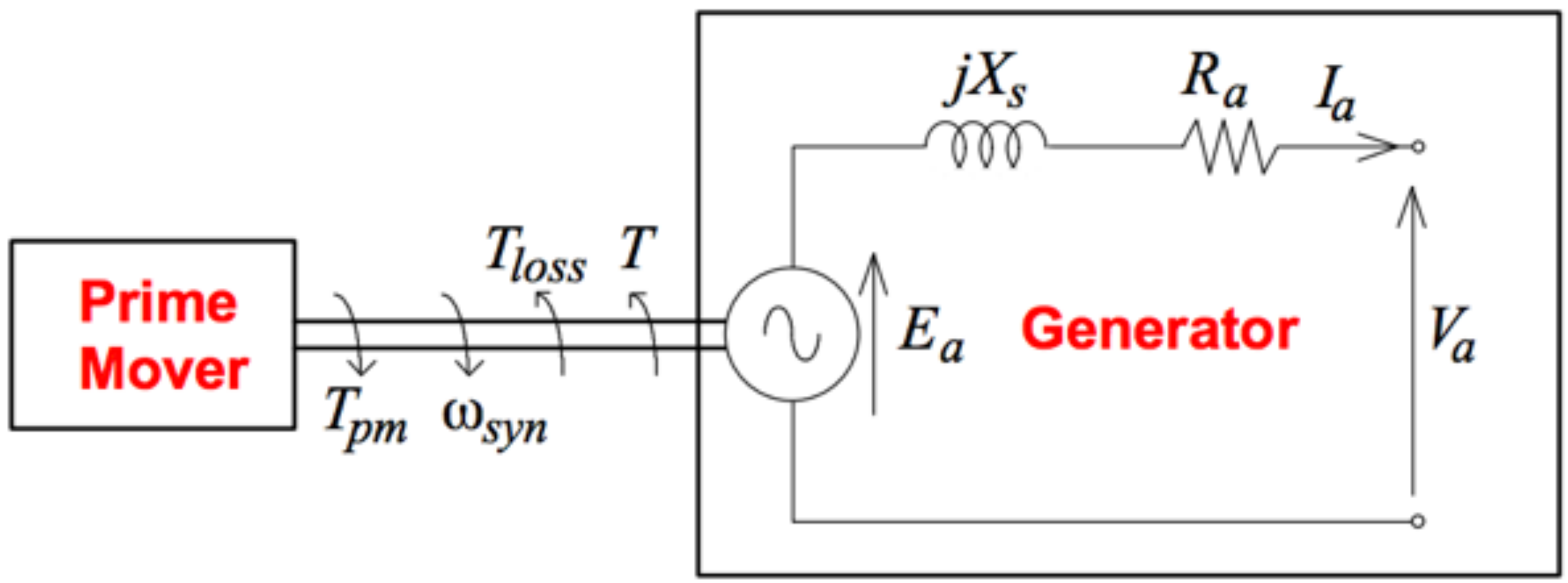
$$0 = S_D - \frac{1}{Z_2}(V_2 - V_3)V_3^*$$

$$\frac{dV_0}{dt} = 0$$

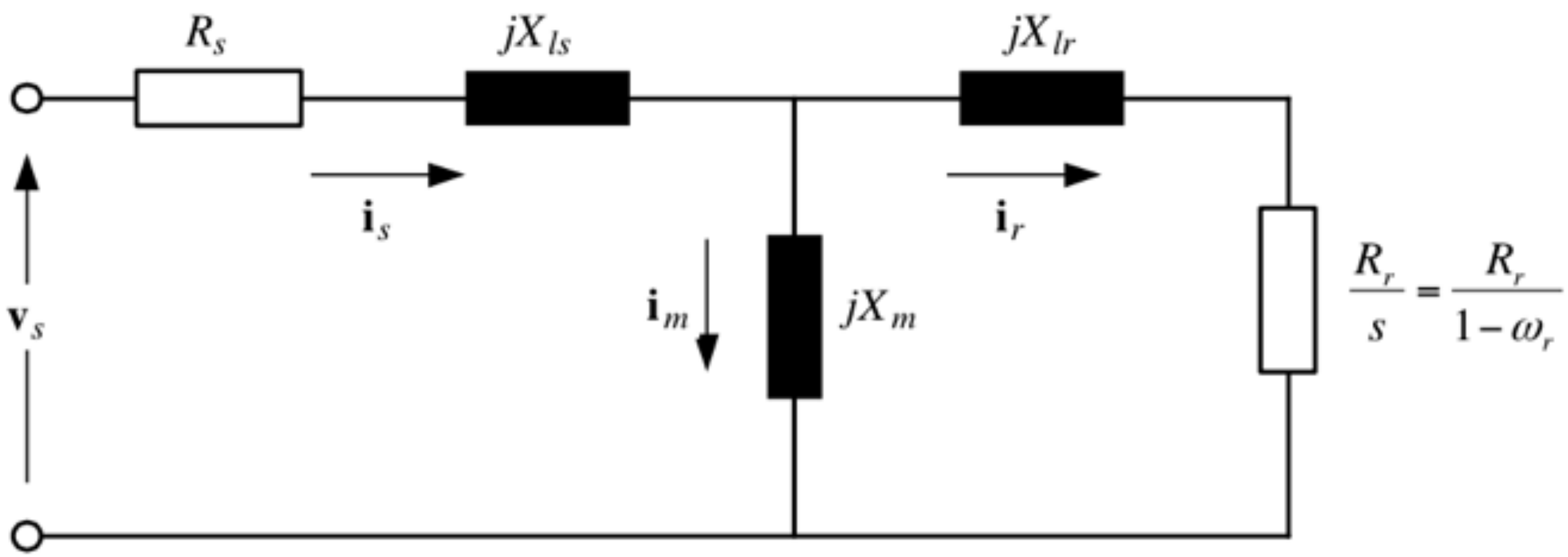
WIND TURBINE



SYNCHRONOUS GENERATOR



TURBINE



$$0 = -V_2 + I_S(R_S + X_S i) + I_R(X_M i)$$

$$0 = I_S(X_M i) + I_R X_R i - I_R \frac{R_R}{w_R/w_S - 1}$$

$$J_G \dot{w}_R = T_M + Imag(I_S(I_S X_S/W_S + I_R X_M/W_S)^*)$$

EMPIRICAL MODELS

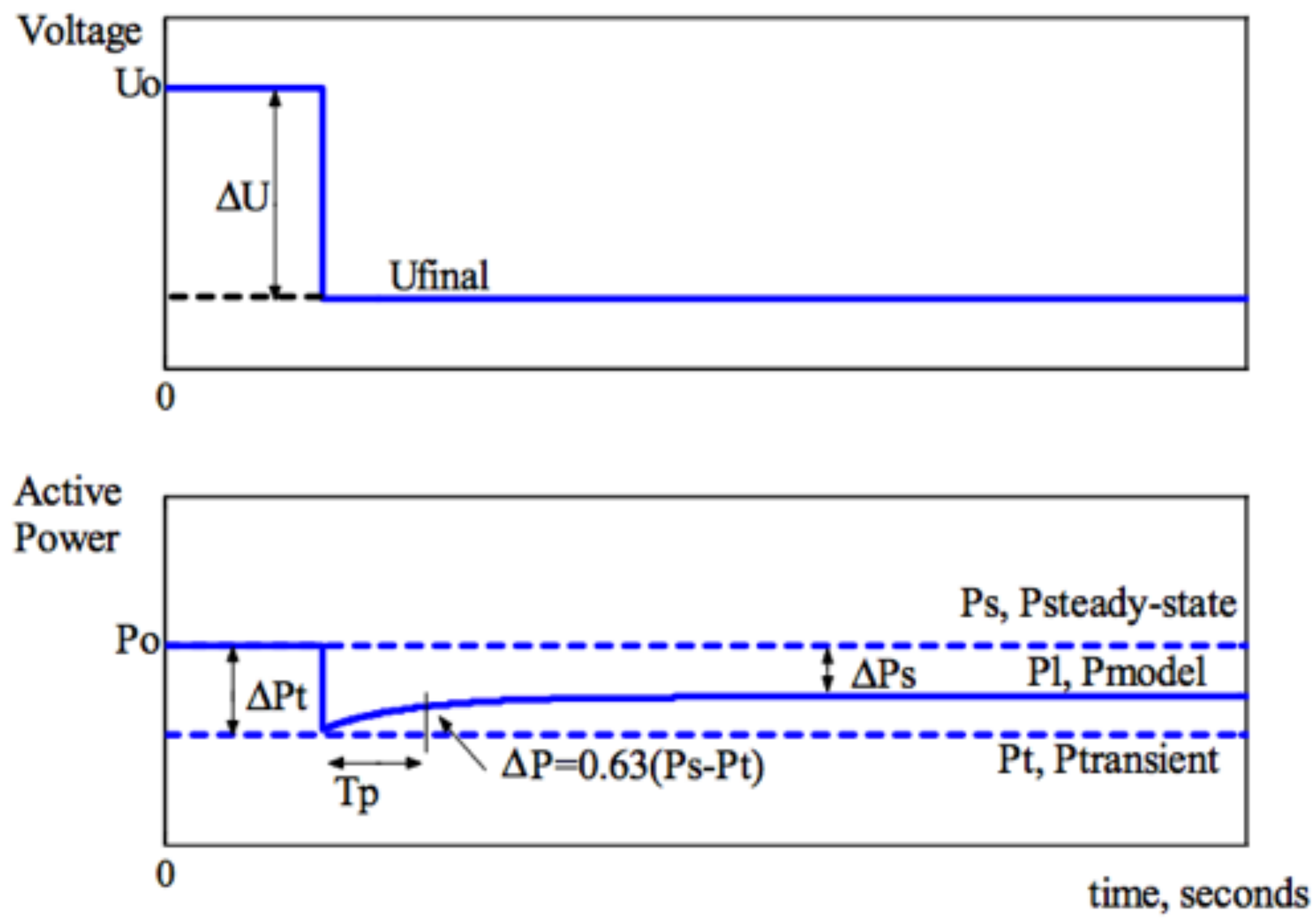


Figure 2.7: Load response under ΔU step, from the Uo-level.

LOAD

$$\dot{x}_D = -\frac{1}{T_P}(P_D - P_S)$$

$$0 = P_D - x_D - P_T|V_3|_2^2$$

$$0 = Q_D$$

TIMERS

$$\dot{x}_{TC} = onC$$
$$y_{TC} = x_{TC} - tresC$$

$$\dot{x}_{TT} = onT$$
$$y_{TT} = x_{TT} - tresT$$

CAPACITOR BANK

$$0 = I_C - (V_2 C n_C) i$$

LOAD FLOW PROBLEM

Real power balance is given by the equation

$$P_i = \sum_{k=1}^{N} |V_i|\,|V_k| \left(Y_{ik}^{R} cos\theta_{ik} + Y_{ik}^{I} sin\theta_{ik}\right)$$

Reactive power balance is given by the equation

$$Q_i = \sum_{k=1}^{N} |V_i|\,|V_k| \left(Y_{ik}^{R} sin\theta_{ik} - Y_{ik}^{I} cos\theta_{ik}\right)$$

Many network problems require the calculation of voltages and currents given the real and reactive powers. This problems is usually solved using the Newton-Rhapson method. The following steps are repeated until convergence is reached.

0) Set all voltage magnitudes to 1.0 p.u. and angles to zero and all voltage magnitudes are set to 1.0 p.u.

1) Solve the power balance equations using the most recent magnitude and angle values $\rightarrow \Delta P, \Delta Q$

2) Linearize the system around the most recent magnitude and angle values $\rightarrow \frac{\partial(P,Q)}{\partial(|V|,\theta)} = J$

3) Solve for the new magnitude and angle values using the mismatch equations.

$$\begin{pmatrix} \Delta|V| \\ \Delta\theta \end{pmatrix} = -J^{-1} \begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix}$$

4) Check the stopping conditions, if met then terminate, else go to step 2.